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Opinion of the scientific achievement of dr Jacek Gulgowski in the habilitation procedure at the University of Gdańsk

The application of dr Jacek Gulgowski to initiate the habilitation procedure at the Faculty of Mathematics and Physics, University of Gdańsk as a scientific achievement indicates the series of six papers entitled "Selected topics in the theory of functions of bounded variation". Among them there are three papers of dr Gulgowski as a single author and three papers written commonly with other co-authors:

[H1] Bugajewski, Dariusz; Gulgowski, Jacek; Kasprzak, Piotr. On continuity and compactness of some nonlinear operators in the spaces of functions of bounded variation. Ann. Mat. Pura Appl. (4) 195 (2016), no. 5, 1513–1530.

[H2] Bugajewski, Dariusz; Gulgowski, Jacek; Kasprzak, Piotr. On integral operators and nonlinear integral equations in the spaces of functions of bounded variation. J. Math. Anal. Appl. 444 (2016), no. 1, 230–250

[H3] Bugajewski, Dariusz; Czudek, Klaudiusz; Gulgowski, Jacek; Sadowski, Jędrzej. On some nonlinear operators in ΛBV-spaces. J. Fixed Point Theory Appl. 19 (2017), no. 4, 2785–2818

[H4] Gulgowski, Jacek. On integral bounded variation. Rev. R. Acad. Cienc. Exactas Fis. Nat. Ser. A Mat. RACSAM 113 (2019), no. 2, 399–422

[H5] Gulgowski, Jacek. Bounded variation solutions to Sturm-Liouville problems. Electron. J. Differential Equations 2018, Paper No. 8, 13 pp.

[H6] Gulgowski, Jacek. Uniform continuity of nonautonomous superposition operators in ΛBV-spaces. Forum Math. 31 (2019), no. 3, 713–726

The co-authors submitted statements about their contributions to papers [H1]-[H3], which certify there was approximately an equal contribution of all authors. The contribution of dr Gulgowski to this papers is 30%, 30% and 25% respectively.

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The series of papers concerns superposition operators in spaces of bounded total variation (which I will further denote as BV). The author considers only one-dimensional definition and develops different generalizations of a classical variation and in a consequence provides corresponding definitions of function spaces. The studies may have an application to various integral equations, e.g. nonlinear Volterra equation or Hammerstein equation. There are also mentioned different applications of the theory such as: image processing or geometric measure theory. Indeed there appear citations to the monographs of Chambolle et al. and Ambrosio et al. Of course both, Chambolle and Ambrosio work in very emerging topics in analysis and their works are important contributions both from the point of view of functional analysis and applications. However what I cannot understand is the relation of these studies with the series of papers, which is presented here. I will use the notation that $f \in BV(\Omega)$ if $f : \Omega \to \mathbb{R}$, $\Omega \subset \mathbb{R}^n$ open set, $f \in L^1(\Omega)$ and the distributional derivative of f is representable by a finite Radon measure in Ω, which means that

$$\int_\Omega f(x)\partial_{x_i}\varphi(x)\, dx = -\int_\Omega \varphi(x) dD_i f \qquad (1)$$

for all $\varphi \in C_c^\infty(\Omega)$, $i = 1, \ldots, n$ and some vector-valued Radon measure

$$Df = (D_1 f, \ldots, D_n f).$$

It is important to underline that the definition of BV space appearing e.g. in the monographs by Chambolle et al. and Ambrosio et al. does not reduce in one-dimensional case to the definition of functions with bounded total variation BV (see Definition 1 of Author's Review). The scope of knowledge sufficient to cope with the problems of function spaces of bounded total variation in one dimension includes within the program of bachelor studies in mathematics. To pass to multi-dimensional problems we need to enter the theory of distributions. I would like to treat in more detail the issue of consistency of modern theory of BV spaces (restricted to one dimension) with one-dimensional theory of functions of bounded total variation.

We can define the variation of f in Ω as follows

$$V(f,\Omega) := \sup\left\{\int_\Omega f(x)\mathrm{div}\varphi(x)\, dx : \varphi \in C_c^1(\Omega), |\varphi| \le 1\right\}. \qquad (2)$$

Then it follows that $f \in BV(\Omega)$ if and only if $V(f,\Omega) < \infty$.

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If however Ω is a bounded interval in $\mathbb{R}$ and $f \in L^1(\Omega)$ and $V(f, \Omega) < \infty$ then there exists a right-continuous function g, such that

$$g = f \ \text{ a.e. in } \Omega$$

and $TV(g) = V(f, \Omega)$, where by TV I understand the total variation according to Definition 1 of Author's Review. Thus in particular if we consider two functions: $f_1(x) = 1$ for $x = 0$ and $f_1(x) = 0$ for $x \in \mathbb{R} \setminus \{0\}$ and the second function $f_1(x) = 2$ for $x = 0$ and $f_1(x) = 0$ for $x \in \mathbb{R} \setminus \{0\}$, then these two objects are not distinguished in the space $\mathcal{BV}(\Omega)$, however in BV these are two different objects.

My general impression of the presented series of papers is that the methods are rather elementary, all the papers follow similar schemes and the author did not convince me that these studies are an important contribution. Some of the statements, which appear as theorems in the papers, could be in fact given to a good student, who finished the course of Analysis or in some cases basic Functional Analysis. For that reason I was not surprised to read in the Author's Review that the recalled facts about compactness are exercises in the monograph of Dunford and Schwarz. The developed theory does not lie in the modern stream of mathematics. The studies arise from the seminal Jordan decomposition of functions of bounded total variations, which allows to present them as a difference of two monotone increasing functions dated to 1881. There is considered impressive number of variations of this standard definition including: $p-$variation, variation in the sense of Young, $\Lambda-$variation, integral variation.

In paper [H1] the authors come back to the result of Morse from 1937 and prove again the continuity of superposition operator in BV spaces (Theorem 2). They claim that the presented proof is shorter and more readable than the one of Morse. The point of writing down a proof of a known fact could have educational purpose, a benefit for students. However having this in mind, I would rather see all the details included, but what is indeed essential here – equivalence between boundedness and continuity of multi-linear forms – is not discussed in much detail. I also do not see the need to formulate and prove Proposition 4, as it is covered by Theorem 2 and the proof follows the same lines up to the need of approximating by polynomials. Further part of the paper is supposed to extend the class of generators, of course paying the cost of weaker topology. In fact the only difference in the proof is a need to use the Hölder inequality. The next theorem in the paper is the second extension of the first fact, this time for the case of $\phi-$variation. Again

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the proof was not surprising. I concentrated more on the proof of existence of solutions to Hammerstein equation, which consists of applying the Schauder fixed point theorem. Here the operator G maps a ball in the space BV_1 into itself. I find the proof not precise. The authors include in the paper definitions which are really standard (e.g. the definition of modulus of continuity), however do not decide to include those, that could be questionable. What is the definition of *completely continuous operator*? Following Conway *A course in functional analysis* (1985) and many others it would mean that for every weakly convergent sequence $\{x^n\}$ from $BV_1(I)$, the sequence $\{G(x^n)\}$ converges in strong topology in $BV_1(I)$. Every compact operator (= mapping every bounded set to a relatively compact one) is completely continuous, but not necessarily vice versa. Also the Schauder fixed point theorem is not stated, which would not be necessary, as this is a standard fact. However that would help to figure out what assumptions the authors claim need to be satisfied for the existence of a fixed point. It happens that in older literature *compact operators* appear to be called *completely continuous* operators. It can also be found that the definition of a *completely continuous* operator is that it is continuous and compact. It would be much better to state the notions precisely.

Paper [H2] deals with the studies on the properties of linear integral operators of Fredholm-type, again the continuity of the operator in the space BV is studied (see Theorem 4 and 5). This result is applied to nonlinear Hammerstein integral equation to show existence of solutions of bounded variation. Again here the compactness is achieved by Helly's selection theorem in the same way as in [H1]. The existence of a fixed point is shown by the Leray-Schauder degree method. In the next subsection the same Hammerstein integral equation is treated again, and the existence of solutions is now shown with the Banach fixed point theorem. The rest of the paper is devoted to the issue of compactness of integral operator when one of the assumptions is weakened. The body of the paper is based on estimates and arguments very similar to those from [H1] and to various papers mentioned in references.

In [H3] again the issue of continuity of superposition operators (here also nonautonomous) is considered, this time in the spaces of functions of bounded $\Lambda-$variation, which is then used to show existence of solutions (having $\Lambda-$variation bounded) to some integral equations. The presented proofs are again not particularly innovative, they mostly follow the reasoning in references [2], [3] or [4] (I refer here to the references in the article). Reading section 5 I am asking myself - how

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many times one can prove existence to Hammerstein integral equation in different spaces? Why not to formulate general conditions on the spaces and operators and then show the existence result. And instead of repeating almost the same proof again and again just to check whether the assumptions are satisfied for various cases. However another question, which immediately appears is - what is the benefit of searching for solutions in a consequent $BV-$type space? The direction is always the following: different concept of spaces is first considered, continuity and compactness of appropriate operators are examined and then, having all necessary tools, it is applied to some integral equation. I do not understand this direction. Where is the place for any physical, biological or engineering motivation for considering the equation with data in a different type of $BV-$space? Has some new real world phenomena motivated the authors to these considerations? Or is it just another paper?

Paper [H4] aims to investigate a notion of functions of bounded variations, however in L^p-setting, to consider a definition which would not be sensitive to a change of a value of function on the set of measure zero. Various concepts of generalizations have been recalled (mostly by Terehin from 70s and 80s), finally the author decides to investigate properties of spaces of functions of bounded $q-$integral $p-$variation. And why the modern notion of BV spaces is not taken into account is a surprise for me. This surely would fit well to the aims of generalization that the author had in mind. The structure of the present paper follows the same scheme as previous ones.

Paper [H5] concerns Sturm-Liouville problem and the studies on relation between existence of solutions to Sturm-Liouville equation and properties of the kernel of appropriate integral operator. The main tool here is Theorem 3.1, which is recalled from [H2]. To prove the main result of the paper, which is Theorem 3.9, the author needs to show that condition (H2) of Theorem 3.1 is satisfied by $k(t, s) = G(s, t)$ with G given by (2.6). This is the main contribution of this paper.

Paper [H6] continues the studies started in [H3, Theorem 4]. Here the local uniform continuity of nonautonomous superposition operators in ΛBV spaces is shown. Although this paper does not fall in the same scheme as papers [H1]-[H4], but still the techniques are really similar, based on standard arguments and estimates.

The list of publications of Jacek Gulgowski contains a series of papers about bifurcation results for Sturm-Liouville equation and other problems. This is the field

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that dr Gulgowski investigated in his doctoral thesis and continued for a couple of years afterwards. Most of the papers from this time were published in average journals. The remaining publications are related with optimization problems in microwave technologies. These results are related with the participation of dr Gulgowski in a common project with the company TeleMobile Electronics (COST). These results, although not very advanced from the view point of mathematical methods, however as claimed by the author in Author's Review had an impact on production technologies. If this is really the case, then I think that such activity is valuable.

Finally I would like to discuss the scientific activity of dr Gulgowski. Doctor Jacek Gulgowski was an assistant at University of Gdańsk in the period 1997 - 2002. For the last 17 years he has been an assistant professor. In the documentation I did not find any information about any longer stays abroad or in other national research units. There is only one stay at Leiden University, which probably resulted in the paper [P18], which is completely different from the other areas studied by dr Gulgowski. The topic is interesting and the studies in this direction could lead to more modern and advanced results. Unfortunately the contribution of dr Gulgowski in this paper is only 10%.

Dr Gulgowski has never headed any international nor national project, however worked as a co-investigator in COST project (realised in collaboration with the company TeleMobile Electronics) and also at one project at the Technical University of Gdańsk. Most of participation in international conferences of Jacek Gulgowski is related with the COST project. Besides of that, he participated in various conferences in Będlewo, Toruń and Słupsk. He was strongly involved in popularisation of mathematics.

Jacek Gulgowski is currently a co-supervisor of a PhD student and for many years taught different courses including: probability theory, algebraic topology, functional analysis, partial differential equations and mathematical software among others.

Conclusion. The presented series of publications is in my opinion not sufficient to be accepted as a scientific achievement in the meaning of Article 16 Paragraph 2 of the Act of 14 March 2003 on Academic Degrees and Title and on Degree and Title in the Field of Art. The scientific value of the presented results is not advanced, and therefore an impact on the development of the discipline is imperceptible. The

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scientific activity of dr Jacek Gulgowski is acceptable, however still rather lower than average. For that reason my overall assessment is negative.

Yours sincerely

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