

# Autopresentation

**Jarosław Korbicz**

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## 1 Name and surname

Jarosław Korbicz

## 2 Diplomas, degrees

16.06.2006 – Ph.D in Natural Sciences, Faculty of Mathematics and Physics, Leibniz Universität Hannover

Dissertation title: "Quantumness of States: From Positive  $P$ -Representations to Entanglement Tests", supervisor: Prof. M. Lewenstein

## 3 Information on previous employment in scientific institutions

- 04.2002-12.2006 Ph.D student, Institut für Theoretische Physik, Leibniz Universität Hannover, Germany
- 01.2007-12.2007 Postdoc, Departament d'Estructura i Constituents de la Matèria, Universidad de Barcelona and Institut de Ciències Fotòniques (ICFO), Barcelona, Spain
- 01.2008-02.2008 Postdoc, Institut de Ciències Fotòniques (ICFO), Barcelona, Spain
- 02.2008-09.2008 Postdoc, Faculty of Applied Physics and Mathematics, Gdańsk University of Technology and Quantum Information Center in Gdańsk (KCIK)
- 10.2008-02.2010 Postdoc, Institute of Theoretical Physics and Astrophysics, University of Gdańsk and Quantum Information Center in Gdańsk (KCIK)
- 02.2010-12.2012 Postdoc, Institut de Ciències Fotòniques (ICFO), Barcelona, Spain
- 01.2013- 12.2014 Assistant professor, Institute of Theoretical Physics and Astrophysics, University of Gdańsk and Quantum Information Center in Gdańsk (KCIK)
- 01.2015 – 04.2015 Scientific research worker, Institute of Theoretical Physics and Astrophysics, University of Gdańsk and Quantum Information Center in Gdańsk (KCIK)

- 05.2015 - Assistant professor, Faculty of Applied Physics and Mathematics, Gdańsk University of Technology and Quantum Information Center in Gdańsk (KCIK)

## 4 Indication of the achievement according to Art. 16.2 Act of laws on academic degrees from 14.03.2003 (Dz. U. nr 65, 595)

### 4.1 Title of the scientific achievement

Single-themed series of publications entitled : *Aspects of quantum-to-classical transition: From entanglement to objectivity*

### 4.2 List of publications

#### References

- [A] J. K. Korbicz, J. Wehr, M. Lewenstein, *Entanglement of positive definite functions on compact groups*, Comm. Math. Phys. **281**, 753 (2008)
- [B] J. K. Korbicz, J. Wehr, M. Lewenstein, *Entanglement and Quantum Groups*, J. Math. Phys. **50**, 062104 (2009)
- [C] J. K. Korbicz, M. Almeida, J. Bae, M. Lewenstein, A. Acin, *Structural approximations to positive maps and entanglement breaking channels*, Phys. Rev. A **78**, 062105 (2008)
- [D] J. K. Korbicz, P. Horodecki, R. Horodecki, *Quantum-correlation breaking channels, broadcasting scenarios, and finite Markov chains*, Phys. Rev. A **86**, 042319 (2012)
- [E] J. K. Korbicz, P. Horodecki, R. Horodecki, *Objectivity in the Photonic Environment Through State Information Broadcasting*, Phys. Rev. Lett. **112**, 120402 (2014)
- [F] R. Horodecki, J. K. Korbicz, P. Horodecki, *Quantum origins of objectivity*, Phys. Rev. A **91**, 032122 (2015)
- [G] J. Tuziemski, J. K. Korbicz, *Dynamical objectivity in quantum Brownian motion*, EPL **112** , 40008 (2015)

[H] J. Tuziemski, J. K. Korbicz, *Objectivisation In Simplified Quantum Brownian Motion Models*, Invited publication outside of the JCR list, *Photonics* **2**, 228 (2015)

[I] J. Tuziemski, J. K. Korbicz, *Analytical studies of Spectrum Broadcast Structures in Quantum Brownian Motion*, *J. Phys. A: Math. Theor.* **49**, 445301 (2016)

### 4.3 Description of the scientific achievement

The scientific achievement comprises the above-mentioned collective publications. My contributions are described in point 1.2 of the annex *List of published scientific papers or creative and professional work and information about teaching achievements, scientific collaboration, and popularization of science*. The contributions of the co-authors are presented in the attached statements.

In what follows, the references cited with letters [A]-[I] indicate the publications belonging to the single-themed series (listed above). The references cited with numbers [1]-[13] refer to other applicant's publications, not belonging to the series. The rest of references is cited with the first author's name and the publication year, e.g. [Schrödinger1935].

#### 4.3.1 Introduction

Despite over 100 years history of quantum theory, quantum-to-classical transition has remained one of the fundamental problems of modern physics [Joos2003]. Its importance comes from the fact that in the variety of its aspects it concerns a whole spectrum of problems: From fundamentals of our understanding of the outside world (e.g. if and what is reality?) to the emerging quantum technologies (e.g. how to tell if a given machine works in a quantum or classical regime?). The amount of controversies and interpretations, developed throughout the years, is huge and the situation resembles in some sense the theory of electromagnetism of moving bodies from the turn of 19th and 20th centuries. The aim of my work was to study various aspects of the quantum-to-classical transition, relying on precise notions and results rather than speculations. One of such notions is the notion of quantum entanglement. It reflects one of the fundamental principles of quantum theory, the linear superposition principle for multipartite systems, and leads to highly non-classical correlations. Noticed already by Schrödinger [Schrödinger1935], entanglement has become in the recent years one of the central objects of both theoretical studies [Horodecki2009] and

various practical implementations. However, its description in the case of realistic, noisy states (so called separability problem) is mathematically a very difficult problem, in some sense still not fully understood, despite the existence of powerful detection algorithms [Doherty2002]. Hence, searching for new methods is justified from both cognitive and practical points of view.

At the other side of the quantum-classical border, in the classical world, entanglement (together with other quantum coherences) is not observed. It is then an interesting aspect what mechanisms lead to its destruction. One of the mathematical representations of such mechanisms are, so called, entanglement breaking channels [Horodecki2003]. These are completely positive, trace-preserving mappings, which when applied to a single subsystem of a multipartite system always lead to disentangled states. The description of such channels has been well known in terms of the, so called, Jamiolkowski isomorphism applied to separable states and measure and prepare procedures. However, it is also known that separable states may still exhibit non-classical correlations [Modi2012] and a question arises to what channels such states correspond.

As I was able to show, there is quite an unexpected connection between this seemingly technical problem and one of the fundamental aspects of the quantum-to-classical transition: The problem of objectivity. As it is well known, an observation (measurement) in quantum theory generically disturbs the state of the observed system. On the other hand, in the world of everyday experience we observe a sort of an observer-independence, described as objectivity. A question arises: If quantum theory is our most basic description of Nature, how does it explain the observed objectivity of the outside world? Pioneering research in this direction has been initiated by W. H. Zurek [Zurek2009], using however information-theoretical notions with not so clear meaning in this context. A deeper analysis, on the fundamental level of quantum states, was needed. The following description of the scientific achievement is structured along the above lines and first presents my studies of entanglement, then the entanglement breaking channels, and finally the emergence of objectivity through information broadcasting during decoherence in open quantum systems.

### 4.3.2 Summary

I started my studies quantum-to-classical transition from mathematical theory of entanglement and the problem of its detection (separability problem). In [A, B], together with the co-authors I developed a novel description of entanglement using the language of harmonic analysis on compact groups

[A] and on their quantum analogs [B]. This was a continuation of my earlier studies before Ph.D [6, 7] and of an earlier work [Gu1985], which however did not study entanglement. The main achievement constitutes the work [A], where we developed a description of quantum states using non-commutative characteristic functions and Peter-Weyl Theorem, formulated the separability problem in this language, and proved a group-theoretical analog of the, so called, Horodecki Theorem [Horodecki1996]. The latter is the central theorem in the entanglement theory, linking the separability problem to the (still fully unsolved) mathematical problem of a general characterization of positive but not completely positive maps. The found group-theoretical analog is, in some sense, its countable-dimensional generalization, unknown before in harmonic analysis. It characterizes separable non-commutative characteristic functions in terms of, so called, positive definiteness preserving maps. A further generalization of parts of the developed formalism to compact quantum groups was made in in work [B]. There, together with the co-authors I formulated an analog of positive-definiteness and Bochner Theorem, characterizing quantum states, defined the notion of separability on compact quantum groups, and proved an analog of the positive partial transpose (PPT) criterion of separability. Those studies have not led so far to new separability criteria as the encountered problems turned out to be new and unknown in harmonic analysis, lacking proper tools to deal with them. However, they opened a way in the opposite direction: To formulate and solve problems in harmonic analysis, using physical methods of entanglement theory.

A further step, after the description of entanglement, were studies, still in an abstract, mathematical way, of how it can be broken, i.e. turned into classical correlations in works [C, D]. In [C], together with the co-authors I studied, so called, structural physical approximations (SPA) [Horodecki2002] of positive but not completely-positive maps – a central object in the entanglement detection theory. Such approximations are possible models of a physical realization of the otherwise unphysical maps. We studied a number of important for the entanglement detection optimal maps [Lewenstein2001], and showed that their physical approximations are entanglement breaking channels and thus allow for conceptually simple implementations in terms of measure and prepare procedures [Horodecki2003]. As a byproduct of those studies, I introduced a novel family of symmetric quantum states: Unitary-symplectic invariant states. This family is quite simple in a description and contains, so called, bound entanglement [Horodecki1998]. I continued my studies of entanglement breaking channels in work [D], where together with the co-authors I introduced and described novel families of such channels,

corresponding to classical-classical (CC) and quantum-classical (QC) correlated states [Modi2012]. We also introduced a notion of, so called, spectral broadcasting, which is a much weaker form of quantum state broadcasting [Barnum1996], where only a spectrum of a state is broadcasted. We also found a quite unexpected and non-trivial connection to the Perron-Frobenius Theory, which shows that every QC channels possesses a whole family of states which it spectrum broadcasts. What is interesting, the measurement which defines the channel does not have to be a von Neumann measurement, but can be a generalized POVM measurement.

The culminating point of my studies of quantum-to-classical transition until now, were the studies of objectivity and its emergence from quantum theory, performed in the works [E, F, G, H, I]. Starting from the pioneering works of W. H. Żurek and collaborators on quantum Darwinism [Zurek2009, Riedel2010], which is an advanced and realistic form of decoherence theory, I applied the idea of spectrum broadcasting from [D] to the problem and introduced together with the co-authors a novel approach, based on, so called, spectrum broadcast structures (SBS) in [E, F]. These are classically correlated states of the system of interest and a part of its environment, reflecting objectivity of a certain observable in the sense of its observer-invariance. In [E], we found these state structures in one of the emblematic models of decoherence and quantum open systems: The illuminated sphere model of Joos and Zeh from 1985 [Joos1985] (see also [Riedel2010] where some form of objectivity was argued). We did so through an introduction of some coarse-graining of the observed photonic environment into, so called, macrofractions, which reflect a macroscopic character of observation. We found the relevant time-scales and, using advanced methods of quantum information theory, derived from SBS the previously used scalar condition of quantum Darwinism. Using Perron-Frobenius Theorem, we also showed a quite surprising possibility of a faithful broadcasting of a certain classical message through the decoherence mechanism. It is worth stressing that the Joos and Zeh model has been an object of an active research for more than 30 years and discovery of a quantitatively new aspect here constitutes a remarkable achievement. The structures found in the concrete model, we were later able to derive in a fully model-independent way in [F]. Its central result consists in establishing a sort of equivalence between an operational notion of objectivity and spectrum broadcast structures. The importance of this result lies in the formulation of a philosophical notion of objectivity in the language of quantum states. What is interesting, the prove relies, among others, on the Bohr's notion of non-disturbance, introduced during the famous Einstein-Podolsky-Rosen vs. Bohr argument of

1935 [EPRB1935]. Quite paradoxically, this notion, then used to defend the quantum, served here to define the classical.

I continued the research on objectivity and spectrum broadcast structures [G, H, I], where together with the co-author I performed an analysis of the quantum Brownian motion (QBM) model [Ullersma1966]. It describes, in the studied case, a central harmonic oscillator linearly coupled to a bath of harmonic oscillators. This is one of the most popular and widely used models of open quantum systems with a rich dynamics, where some form of objectivization was already argued in [Blume-Kohout2008]. Despite the fact that the model is described by a simple Hamiltonian, quadratic in the canonical variables, finding a solution for only partially reduced state, describing together with the central oscillator also a part of its environment, turned out to be a difficult task due to a lack of proper tools. Working in the recoilless limit, adequate to the posed questions, we found in [G] a novel type of state structure in this model – a dynamical spectrum broadcast structure. Its characteristic feature is its time dependence. It encoded the initial position of the central oscillator, as well as some other characteristics of its motion (e.g. the frequency). Those studies were continued in [H] (this is an auxiliary publication outside the JCR list, which nevertheless contains interesting results) in simplified regimes of the model, using some form of an ergodic theorem for almost periodic functions. We introduced there a notion of macroscopic objectivity – objectivity emerging only on macroscopic scales. In [I], the numerical analysis of [G] was completed with analytical studies. We derived conditions of the SBS formation in terms of the model parameters such as the temperature, time, and the observed macrofraction size in both low and high temperature regimes.

One can look at the above research from the perspective of open quantum systems. Similarly to quantum Darwinism, it breaks the standard paradigm concerning the role of the environment as a source of noise only, and poses a new set of questions, e.g.: What information about the system is gathered by the environment, how is this information reflected in the state structure including a part of the environment, and is this structure close to SBS? Answers to those questions can help revealing new aspects of quantum-to-classical transition in many-body systems and may be useful on both the technological level (e.g. helping to control an "information leakage" in quantum registers) and on the fundamental one (e.g. in studies of the quantum measurement problem or attempts to derive the space-time as a quantum effect). A big challenge here is, however, a lack of proper tools for generating solutions and it is inevitable to employ the whole spectrum of available methods (e.g. path integrals) for developing them.



## 5 Other scientific achievements

### Bibliometric data:

- number of publications **22** (**14** after Ph.D)
- total number of citations: **385** (**364** without self-citations)
- Hirsch-index: **9**
- total impact factor: **75**

### 5.1 Before Ph.D

#### 5.1.1 General Relativity

We studied Bondi-Sachs metrics adapted to a null foliation of spacetime in [1]. We wrote down the Hilbert action for such metrics and applied the Dirac procedure of treating the constraint systems to obtain the corresponding Hamiltonian formalism, which is an analog of the famous ADM formalism [Arnowitt1959] but with a null foliation.

#### 5.1.2 Studies of state quantumness using generalized squeezing

Together with the co-authors I gave a series of conditions characterizing non-classical states of continuous variable (CV) systems [2] and systems of many spins  $1/2$  [3, 4] in terms of generalized squeezing. For CV systems, we derived novel conditions for the existence of a positive  $P$ -representation and connected them to the Hilbert's 17th problem (existence of positive polynomials which are not sum of squares of other polynomials). In case of spin systems, we studied symmetric states and derived conditions for two- and three-body entanglement in terms of generalized spin squeezing for the macroscopic spin [3]. These conditions were then used for a post-processing of experimental data in [4], confirming a non-classical character of the obtained states.

#### 5.1.3 Fermi-Dirac statistics and number theory

In [5] we studied a connection between the Fermi-Dirac statistics for an ideal Fermi gas in a harmonic trap and the partition problem of a natural number. Using methods of quantum statistical mechanics, we derived analytical expressions for cumulants of the probability distribution of the number of different partitions.

#### 5.1.4 Application of harmonic analysis to the studies of quantumness

Together with the co-author I proposed in [6] a novel approach to entanglement description in finite-dimensional systems, based on compact groups and harmonic analysis on them. We formulated basic theorems, characterizing entangled states in terms of their non-commutative characteristic functions. I further developed this method in later publications. Generalizing the approach to the (non-compact) Heisenberg-Weyl group, we developed in [7] a novel description of quantum-to-classical transition using non-commutative characteristic functions instead of Wigner functions.

### 5.2 After Ph.D

#### 5.2.1 Entanglement, algebraic geometry, and statistical mechanics

Looking for novel methods to describe entanglement, we proposed in [8] a description using basic algebraic geometry – a study of zeros of a set of polynomials. The corresponding polynomials were constructed using: 1) So called Stiefel manifolds, describing the set of all convex decompositions of a given density matrix; 2) so called, Segre embedding of projective spaces  $\mathbb{C}P^m \times \mathbb{C}P^n \rightarrow \mathbb{C}P^{mn}$  (closely related to the generalized quantum concurrence). In this way, the common zeros of the above polynomials correspond to a product decomposition of a given density matrix, i.e. its separability. A lack of further algebraic-geometrical tools for studying the obtained system, forced a physical approach to the problem: An application of statistical-mechanical methods. By defining a constrained Hamiltonian system and introducing an artificial temperature (as a regularizing parameter), we connected with the original problem a statistical mechanical model. Its further studies were performed numerically on the example of Werner states to test the method. Although some sign of entangled-separable transition was observed, the needed numerical resources turned out to be substantial.

#### 5.2.2 Non-signaling correlations and Bell inequalities

Understanding why Nature is not more non-local than quantum theory has recently risen to one of the most fundamental problems of quantum theory (see e.g. [Pawlowski2009]). This problem can be formulated using only a minimal description of the underlying theory in terms of conditional probabilities  $p(a_1, \dots, a_n | X_1, \dots, X_n)$ , representing joint distribu-

tions of measurement results  $a_1, \dots, a_n$  of some (abstract) measurement procedures  $X_1, \dots, X_n$ , applied by  $n$  observers. Finite speed of information transmission puts a set of linear constraints on those probabilities (so called non-signaling conditions), cutting some convex polytope. However, quantum-mechanical correlations form a proper subset of this polytope. Finding conditions which would uniquely characterize the quantum set, and would thus provide an answer to the initial question, turned out to be a very difficult task. I took part in such a research in [9], where we showed that simultaneously assuming: 1) non-signaling and 2) description of local measurements using quantum formalism, leads to correlations more general than quantum. The result is quite surprising as in order to obtain it one has to go to three or more parties – for two parties non-signaling and local quantum mechanics reproduce all of the quantum correlations [Barnum2010]. Our work also connected the problem of quantum correlations with multipartite generalizations of the Gleason theorem.

It turned out that the constructed example is based on a very interesting object – unextendible product bases (UPB's). These are orthonormal sets of vectors in multipartite Hilbert spaces, consisting solely of product vectors and such that one cannot find any product vector orthogonal to the subspace spanned by the set. These are important objects in the theory of entanglement, which allow to construct so called entanglement witnesses (which we actually dealt with). This observation suggested a connection between UPB's and Bell inequalities, which are sets of conditions describing classically allowed correlations. We studied this connection in [10], where we developed procedures to construct: 1) families of Bell inequalities from UPB's and 2) UPB's from some Bell inequalities. The obtained Bell inequalities turned out to be quite interesting as they were broken only by general non-signaling correlations, with quantum and classical bounds being equal.

### 5.2.3 Superadditivity of Multiple Access Quantum Channels

In [11, 12] we showed how entanglement allows to increase the classical capacity of quantum multiple access channels (MAC's) beyond the classically allowed region. Concentrating on continuous-variable Gaussian channels, we studied a data transmission protocol, inspired by the quantum dense coding, and gave its non-classical regimes. What is interesting, in those regimes one of the senders formally sends a constant symbol, thus transmitting no information, but nevertheless helps the other sender increase his/her capacity. The found protocol constitutes a CV analog of the discrete-variables one, found earlier [Czekaj2009]. The proposed scheme turned out to be

conceptually simple and hence very interesting from an application point of view – what was needed were beam splitters and a source of a two-mode squeezed light. However the required level of squeezing was high at 6dB, which nevertheless is within the range of the state-of-the art experiments.

#### 5.2.4 Typical equilibration time-scales in thermodynamical systems

One of the fundamental problems of statistical mechanics and thermodynamics is explanation of the thermalization phenomenon (see e.g. [Popescu2006]). One of the aspects of this problem are the observed time-scales. Various versions of the ergodic theory and recurrence theorems do not provide an answer for those type of questions and different methods are needed. I took part in a novel research on this problem in [13], where we switched from a fixed to a random Hamiltonian and asked questions about typicality. Working in the finite-dimensional setting, the total system+environment Hamiltonian was assumed to be random and drawn from an ensemble, described by the Haar measure (the eigenbasis) and independent Gaussian measures (the energies). The main object of the study was a Hilbert-Schmidt distance between the reduced state of the system and its asymptotic, decohered in the energy basis. Randomizing the Hamiltonian allowed to derive typical time-scales of the decay of this norm. Of course this still does not imply the thermalization as the equilibrium state need not be a Gibbs state, but nevertheless constitutes an interesting contribution to the problem of thermalization, appreciated by specialists. I am further developing the methods introduced in this work, applying them to studies of typicality of objective properties in quantum mechanics.

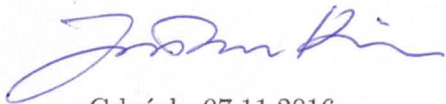
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