Summary of dissertation: Permutation graphs

Monika Rosicka

In this dissertation we study the properties of various kinds of permutation graphs.

Chapter 2 is devoted to prism graphs. A prism graph πG for a given undirected graph $G = (V, E)$ and permutation $\pi : V \mapsto V$ is a graph obtained from two copies $G = (V, E)$ and $G' = (V', E')$ of G by adding a matching defined by the permutation π. For such graphs we study several parameters related to domination.

Sections 2.2 and 2.3 focus on the domination number γ of a prism graph, particularly the universal fixer conjecture, which states that the only universal fixers, i.e. graphs satisfying $\gamma(\pi G) = \gamma(G)$ for every permutation π, are edgeless graphs $\overline{K_n}$. Section 2.2 contains the proof that no graph with a C_3-free vertex is a universal fixer. In section 2.3 we present a complete proof of the conjecture.

In section 2.4 we study several other types of domination. we present some observations on the paired, total, connected, convex and weakly convex domination numbers of πG. In particular, we characterize convex prism fixers, i.e. graphs G such that $\gamma_{con}(G) = \gamma_{con}(IdG)$, and convex prism doublers, i.e. graphs for which $2\gamma_{con}(G) = \gamma_{con}(IdG)$, as well show that for every k there exist graphs G, H and permutation $\pi : V(G) \mapsto V(G)$ and $\sigma : V(H) \mapsto V(H)$ such that $\gamma(G) - \gamma(\pi G) \geqslant k$ and $\gamma(\sigma H) - 2\gamma(H) \geqslant k$.

In Chapter 3 we describe permutation graphs G over H, denoted by $G \bowtie^{\Pi} H$ which are a generalizaton of both prism graphs and Cartesian products of graphs. we give the bounds on the domination number of $G \bowtie^{\Pi} H$ as well as make some observations on the minimum dominating sets of such graphs.

In Chapter 4 we study the properties of labeled graphs. A labeled graph (G, K) is a graph G with an edge-labeling $K : E(G) \mapsto S_n$ which assigns permutations of the set $[n] = \{0, ..., n-1\}$ to the edges of G. For a given labeled graph (G, K) we consider vertex-assignments $k : V(G) \mapsto [n]$. A contradiction in a vertex-assignment k is an edge uv such that $\pi_{uv}(k(u)) \neq k(v)$, where π_{uv} is the permutation assigned to the edge uv. we study the number of assignments without contradictions as well as the minimum number of contradictions over all possible assignments. For this purpose we introduce the permutation graph $KG = \overline{K_n} \bowtie^K G$. we also consider labeled graphs as a generalization of signed graphs and define labeled graph equivalence as a generalization of signed graph equivalence.

In section 4.7, we describe the quantum-mechanical interpretation of some of the findings in Chapter 4. we summarize some results from *Linear game non-contextuality and Bell inequalities - a graph-theoretic approach*, in which labeled graphs are applied in the study of contextuality.