Tomasz Paterek

Autopresentation

2015

I. NAME AND SURNAME

Tomasz Paterek

II. DIPLOMAS, DEGREES

24.05.2007 – Ph.D in physics, Faculty of Mathematics, Physics and Informatics, University of Gdańsk. Dissertation title: "Quantum communication". Supervisor: Prof. Marek Żukowski.

III. INFORMATION ON PREVIOUS EMPLOYMENT IN SCIENTIFIC INSTITUTIONS

- 01.04.2007-31.08.2008, Junior Scientist, Institute for Quantum Optics and Quantum Information, Austrian Academy of Sciences, Austria
- 03.11.2008 - 01.05.2012, Research Fellow, Centre for Quantum Technologies, National University of Singapore, Singapore
- 17.07.2012 - 16.07.2015, Research Assistant Professor, Centre for Quantum Technologies, National University of Singapore, Singapore
- Since 17.07.2012: Assistant Professor, Division of Physics and Applied Physics, School of Physical and Mathematical Sciences, Nanyang Technological University, Singapore

IV. INDICATION OF THE ACHIEVEMENT ACCORDING TO ART. 16.2 ACT OF LAWS FROM 14 MARCH 2003 R. ON ACADEMIC DEGREES (DZ. U. NR 65, 595)

A. title of the achievement

Single-themed series of publications entitled *Nonclassical correlations in quantum systems: measures, phenomena and applications.*

B. List of publications

[A] P. Badziąg, Č. Brukner, W. Laskowski, T. Paterek, M. Żukowski, *Experimentally friendly geometrical criteria for entanglement*, Phys. Rev. Lett. **100**, 140403 (2008).

[B] K. Modi, T. Paterek, W. Son, V. Vedral, M. Williamson, *Unified view of quantum and classical correlations*, Phys. Rev. Lett. **104**, 080501 (2010).

[C] W. Laskowski, T. Paterek, Č. Brukner, M. Żukowski, *Entanglement and communication-reducing properties of noisy N-qubit states*, Phys. Rev. A **81**, 042101 (2010).

[D] P. Kurzyński, T. Paterek, R. Ramanathan, W. Laskowski, D. Kaszlikowski, *Correlation complementarity yields Bell monogamy relations*, Phys. Rev. Lett. **106**, 180402 (2011).

[E] W. Laskowski, D. Richart, C. Schwemmer, T. Paterek, H. Weinfurter, *Experimental Schmidt decomposition and state independent entanglement detection*, Phys. Rev. Lett. **108**, 240501 (2012).

[F] T. K. Chuan, J. Maillard, K. Modi, T. Paterek, M. Paternostro, M. Piani, *Quantum discord bounds the amount of distributed entanglement*, Phys. Rev. Lett. **109**, 070501 (2012).

[G] K. Modi, A. Brodutch, H. Cable, T. Paterek, V. Vedral, *The classical-quantum boundary for correlations: discord and related measures*, Rev. Mod. Phys. **84**, 1655 (2012).

[H] W. Laskowski, M. Markiewicz, T. Paterek, R. Weinar, *Entanglement witnesses with variable number of local measurements*, Phys. Rev. A **88**, 022304 (2013).

[I] A. Fedrizzi, M. Zuppardo, G. G. Gillett, M. A. Broome, M. de Almeida, M. Paternostro, A. G. White, T. Paterek, *Experimental distribution of entanglement with separable carriers*, Phys. Rev. Lett. **111**, 230504 (2013).

[J] T. K. Chuan, T. Paterek, *Separable states improve protocols with finite randomness*, New J. Phys. **16**, 093063 (2014).

[K] C. Schwemmer, L. Knips, M. C. Tran, A. de Rosier, W. Laskowski, T. Paterek, H. Weinfurter, *Genuine multipartite entanglement without multipartite correlations*, Phys. Rev. Lett. **114**, 180501 (2015).

C. Description of the scientific achievement

The scientific achievement comprises above-mentioned collective publications. My contribution is described in point I.B of the annex *List of published scientific papers or creative and professional work and information about teaching achievements, scientific collaboration and popularisation of science.* The contribution of coauthors is presented in the attached statements.

The references cited with letters, e.g. [A], refer to the publications that belong to the series (listed above). The references cited with numbers, e.g. [1], refer to the applicant's publications that do not belong to the series. Other references are cited with the name of the first author and the year of publication, e.g. [Einstein1935].

1. Introduction

Quantum systems can be correlated in ways inaccessible to classical objects. Various notions of classicality of correlations exist and within this scientific achievement we classify them, quantify them and study their consequences. For example, one may regard as classical the local realistic world view put forward by Einstein, Podolsky and Rosen [Einstein1935]. Using modern language this is the world in which the results of experiments can be calculated by local algorithms supplied with data transmitted no faster than the speed of light. Bell showed that correlations between outcomes of such local programs are bounded [Bell1964], and there exist entangled quantum states with correlations violating this bound. Interestingly, Werner proved that there are other entangled quantum states that generate outcomes in perfect agreement with a local realistic view [Werner1989]. Therefore according to local realism even correlations generated by some entangled states are classical.

Clearly one can object that the notion of local realism is too broad as it is also present in models different from classical physics. The set of states admitting a local realistic model is reduced if another notion of classicality is introduced. One may regard as classical those states which can be prepared with the help of local operations and classical communication (LOCC). According to this notion, the set of classical states is exactly the set of separable (not entangled) quantum states [Horodecki2009], and quantum correlations correspond exactly to entanglement. However, one may object to this notion of classicality too, having in mind the nature of the operations allowed in the framework of LOCC. Indeed, local operations here allow for the preparation of indistinguishable pure quantum states, whereas it is impossible to prepare pure indistinguishable states of a classical bit: a classical bit about which we have full knowledge (in a pure state) can be either in state "0" or in state "1", i.e., in one of two fully distinguishable states. General quantum states which satisfy this final classicality constraint form a subset of the separable quantum states and accordingly define some separable states as quantumly correlated. In this spirit the present scientific achievement first discusses phenomena related to local realism, then talks about entanglement/separability border and finally focuses on quantum discord and related notion of classical correlations (discord identifies as classically correlated states that form a subset of separable states).

2. Summary

We begin with the most relaxed notion of classicality in our hierarchy, i.e. the notion of local realism. It is well known that correlations admitting local realistic description satisfy Bell inequalities, and furthermore that correlations of quantum mechanically entangled states may violate those inequalities. However, it turns out that simultaneous violation of more than one Bell inequality is often not possible. The name "Bell monogamy" was coined for this effect as in the simplest case maximal violation of one Bell inequality implies no violation for another one. In Ref. [D] we show that quantum bounds on violations of multiple (multipartite) Bell inequalities can be derived from a relation we call correlation complementarity. Correlation complementarity is a version of uncertainty relation applied to correlations between multiple quantum systems. Additionally to explaining many tight Bell monogamy relations, we showed that for multipartite Bell inequalities it is actually possible to violate simultaneously more than one Bell inequality. Correlation complementarity has also found other applications, e.g. in entanglement detection [E,30] and quantum-to-classical transition [22].

Our second notion of classicality is separability. We propose a new complete characterisation of quantum entanglement in terms of directly experimentally accessible correlation functions [A]. This is then used to derive new experimentally friendly entanglement detection techniques [C,E,H] as well as to study the interplay between entanglement and local realism [C]. Our methods were successfully implemented in the quantum optics laboratory of Prof. Weinfurter [E]. Although quantum entanglement manifests itself in correlations, and in bipartite setting entanglement is always supported by existence of bipartite correlations, in the multipartite case genuinely multiparty entangled states exist that do not give rise to any multiparty correlations [Kaszlikowski2008]. We showed that for any pure state (of odd number of qubits) there exists an "anti-state" with exactly opposite multipartite correlations. Thus taking an even mixture of these pure states results in states without multipartite correlations. Many of them are genuinely multiparty entangled as demonstrated theoretically and experimentally in Ref. [K].

Finally we turn our attention to quantum discord, which decides as classical correlations of states that can be locally measured without disturbing them. We introduce a unified approach which allows for direct comparison of entanglement and discord as it uses the same mathematical quantities for both [B]. Namely, the amount of entanglement is measured by relative entropy from the closest separable state and amount of discord is measured by the same quantity calculated to the closest classical state. It turns our that

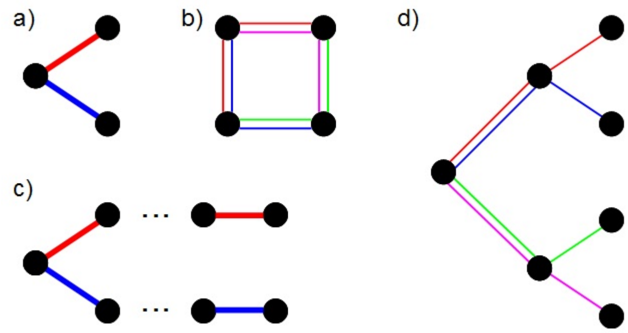

FIG. 1 Bell monogamy scenarios. a) Three observers (vertices) are trying to violate two bipartite Bell inequalities (edges). b) Very compressed scenario where only four observers try to violate four tripartite Bell inequalities. c) and d) are yet other exemplary possibilities for which we provide tight Bell monogamy relations.

a new type of correlations appears naturally in this formulation and describes non-classical correlations that exclude entanglement, i.e. in the closest separable state to the original state. We show that such introduced relative entropy of discord is of utmost importance in entanglement distribution protocols as its value measured on the system communicated in the protocol bounds the amount of distributed entanglement [F]. We emphasise that entanglement gain is not bounded by the communicated entanglement, but by the communicated discord which can be positive even in separable states. In this way we identify discord as necessary resource for entanglement distribution and the quantity which empowers entanglement gain via separable carriers noted in [Cubitt2003]. The latter effect was for the first time observed in Ref. [I] as well as parallel works [Vollmer2013,Peuntinger2013]. Other applications of discord include its role in communication and computing scenarios (example of random access codes is treated in Ref. [J]) and are summarised in our review [G].

We now describe the works belonging to the series in more detail.

3. Local realism

Local realism asserts that all possible measurements have simultaneously well-defined results that only depend on (local) parameters of measuring devices and particles that enter them. Special relativity is an example of a local realistic theory because given positions and momenta of a system of particles we can in principle calculate the measurement results for all possible physical quantities. Furthermore, space-like separated events cannot influence each other imposing locality. Since quantum mechanics gives only probabilistic predictions, it was puzzling already to the fathers of the theory whether a deeper local realistic theory exists where quantum probabilities originate form the lack of knowledge of some of its variables. Bell was the first one to show that such a local hidden variable theory underlying quantum predictions does not exist [Bell1964]. He derived an inequality that is satisfied by all correlations of local realistic theories but is violated by certain quantum correlations. This violation became known as "quantum non-locality" though one should be careful to note that quantum mechanics does not allow for superluminal communication (so called no-signalling principle). This peaceful coexistence of quantum mechanics and special relativity can be harnessed to make statements about quantum predictions themselves.

Bell monogamy relations. Consider as an example the situation in Fig. 1a). Three observers (black vertices) are trying to violate two Bell inequalities (colourful edges) such that the measurement results of the left observer enter into both Bell inequalities. It turns out that if it were possible to violate both inequalities, left observer could communicate instantaneously with the right observers breaking no-signalling principle [Pawlowski2009]. However, no-signalling alone does not recover exact quantum bounds on the amount of violations [Toner2006]. In Ref. [D] we recover those bounds from the relation we called correlation complementarity.

Correlation complementarity is a form of uncertainty relation: it reveals trade-offs in expectation values measured in arbitrary quantum state. For dichotomic ± 1 observables with corresponding anti-commuting operators, A_k, we have [D, Wehner2008, Wehner2010]:

$$\sum_k \alpha_k^2 \leq 1, \tag{1}$$

where α_k is the expectation value of A_k. Therefore, if one of the expectation values is big, say close to one, the other ones have to be small, giving rise to the trade-off. We now consider complete set of correlation Bell inequalities for many observers, each choosing one of two measurement settings and with dichotomic measurement outcomes [Werner2001,Zukowski2002]. Ref. [Zukowski2002] derives the upper bound on the quantum value of the general Bell operator $\mathcal{B}$ with the local realistic bound normalised to one:

$$\mathcal{B}^2 \leq \sum_{j_1 \dots j_N = x,y} T_{j_1 \dots j_N}^2, \tag{2}$$

where summation is over orthogonal local directions x and y which span the plane of the local settings, and $T_{j_1 \dots j_N}$ are the elements of the so-called correlation tensor, which gives alternative to density matrix description of quantum states. Namely, any state of N qubits with density matrix ρ can also be written as

$$\rho = \frac{1}{2^N} \sum_{\mu_1 \dots \mu_N = 0}^{3} T_{\mu_1 \dots \mu_N} \sigma_{\mu_1} \otimes \dots \otimes \sigma_{\mu_N}, \tag{3}$$

where σ_0 is the identity matrix and $\sigma_1, \sigma_2, \sigma_3$ denote Pauli matrices. Our method for finding quantum bounds for Bell violations is to use condition (2) for combinations of Bell parameters and then identify sets of anti-commuting operators in order to utilise inequality (1).

As a warm up let us derive the Tsirelson bound [Tsirelson1980]. For two qubits the general Bell parameter is upper bounded by $\mathcal{B}^2 \leq T_{xx}^2 + T_{xy}^2 + T_{yx}^2 + T_{yy}^2$. One can identify here two vectors of averages of anti-commuting observables, e.g. (T_{xx}, T_{xy}) and (T_{yx}, T_{yy}). Applying correlation complementarity (1) to each of these vectors we find $\mathcal{B} \leq \sqrt{2}$, which is exactly the Tsirelson bound (recall that the local realistic bound of $\mathcal{B}$ is here fixed to one).

The monogamy relation of Fig. 1a) is obtained as follows. Consider the sum of two Bell parameters and use (2) to upper bound each parameter individually: $\mathcal{B}_{AB}^2 + \mathcal{B}_{AC}^2 \leq \sum_{k,l=x,y} T_{kl0}^2 + \sum_{k,m=x,y} T_{k0m}^2$. It is important to note that the settings of A are the same in both sums and so are orthogonal directions x and y. This allows us to arrange the Pauli operators corresponding to correlation tensor components entering the sums into the following two sets of anti-commuting operators: $\{\sigma_x\sigma_x\sigma_0, \sigma_x\sigma_y\sigma_0, \sigma_y\sigma_0\sigma_x, \sigma_y\sigma_0\sigma_y\}$ and $\{\sigma_y\sigma_x\sigma_0, \sigma_y\sigma_y\sigma_0, \sigma_x\sigma_0\sigma_x, \sigma_x\sigma_0\sigma_y\}$, where the order gives the qubit on which the operator acts. Since we were able to identify just two such groups, the correlation complementarity gives $\mathcal{B}_{AB}^2 + \mathcal{B}_{AC}^2 \leq 2$. Accordingly, once one Bell inequality is violated, the other one has to be satisfied.

Using the same method we derive Bell monogamy relations related to more complicated graphs, examples given in Fig. 1. They reveal that for multipartite Bell inequalities it is actually possible to violate more that one inequality at a time, but there is still a form of monogamy. This is nicely illustrated by the inequality corresponding to Fig. 1b). It turns out that the monogamy relation reads:

$$\mathcal{B}_{ABC}^2 + \mathcal{B}_{ABD}^2 + \mathcal{B}_{ACD}^2 + \mathcal{B}_{BCD}^2 \leq 4. \tag{4}$$

Therefore, in principle even three Bell inequalities could be violated, but not four. Indeed we also proved that all the relations derived in this way are tight, i.e. all mathematically allowed values for the Bell parameters can be realised by suitable measurements on suitable states.

4. Separability

We turn our attention to probably best known kind of nonclassical correlations — quantum entanglement. States which are not entangled are called separable and can be written in the following mathematical form:

$$\rho_{\text{sep}} = \sum_i p_i \rho_i^{(1)} \otimes \cdots \otimes \rho_i^{(N)}, \tag{5}$$

where p_i's are probabilities and $\rho_j^{(n)}$ is an arbitrary state of the nth subsystem. We provide alternative characterisation of entanglement, show how it translates to experimentally useful expressions and study effect of "entanglement without correlations".

Necessary and sufficient condition. Let us begin with a simple geometrical fact: if a scalar product of two real vectors $\vec{s}$ and $\vec{e}$ satisfies $\vec{s} \cdot \vec{e} < \vec{e} \cdot \vec{e}$, then $\vec{s} \neq \vec{e}$. We adopt it to entanglement detection by replacing the scalar product with inner product and taking as vectors correlation tensors (or density matrices) of separable and entangled states:

$$\max_{T^{\text{sep}}} (T, T^{\text{sep}}) < (T, T) \quad \Longrightarrow \quad T \text{ is entangled.} \tag{6}$$

Since T^{sep} is a convex mixture of correlation tensors of product states, we can introduce simpler condition:

$$\max_{T^{\text{prod}}} (T, T^{\text{prod}}) < (T, T) \quad \Longrightarrow \quad T \text{ is entangled.} \tag{7}$$

By fixing the inner product to $(A, B) = \sum_{j_1 \dots j_N = 1}^{3} A_{j_1 \dots j_N} B_{j_1 \dots j_N}$, we note that the left-hand side of our criterion is just the maximal possible correlation function of the state, $T_{\max}$, and the criterion becomes:

$$T_{\max} < (T, T) \quad \Longrightarrow \quad T \text{ is entangled.} \tag{8}$$

Although very simple this criterion is optimal in some nontrivial cases. Consider Werner state of two qubits $\rho = p \left| \psi^- \right\rangle \left\langle \psi^- \right| + (1 - p) \frac{1}{4} \mathbb{1}$, where $\left| \psi^- \right\rangle$ is the Bell singlet state and $\frac{1}{4} \mathbb{1}$ describes white noise. It is easy to verify that non-vanishing correlations of this state are $T_{xx} = T_{yy} = T_{zz} = -p$. Thus, $T_{\max} = p$ while $(T, T) = 3p^2$. Our criterion shows that this state is entangled for $p > 1/3$, i.e. exactly for all entangled states of the family.

In general this criterion is of course not optimal and we extended it to a necessary and sufficient condition for entanglement [A]. It turns out that one has to consider generalised inner product defined via a positive semidefinite metric G:

$$(A,B)_G = \sum_{\mu_1\dots\mu_N,\nu_1\dots\nu_N=0}^{3} A_{\mu_1\dots\mu_N} G_{\mu_1\dots\mu_N;\nu_1\dots\nu_N} B_{\nu_1\dots\nu_N}. \tag{9}$$

Then the following condition becomes necessary and sufficient for entanglement:

$$\exists_G \max_{T^{\text{prod}}} (T, T^{\text{prod}})_G < (T,T) \quad \iff \quad T \text{ is entangled.} \tag{10}$$

The implication from left to right is clear from the argument above. For the converse statement, the (extended) correlation tensor of an entangled state has to lie at a finite distance from a set of separable correlation tensors (this translates into the strict inequality in (10)). Since the set of separable correlation tensors is convex, it contains the closest tensor T_0 to our tensor of interest T. The metric for which the left hand side holds can be symbolically expressed as a projector onto T_0 [A]. We also showed that entanglement witnesses [Horodecki1996] correspond to metrics that can be cast as projectors. Since more general metrics are allowed in our criterion, it is strictly reacher than the family of entanglement witnesses.

Detection of quantum entanglement. The usefulness of this criterion comes from the fact that it involves directly measurable correlations. There is no need to process them in any way or reconstruct a density matrix. Another advantage comes from considering the simplest criterion of the family, i.e. condition (8) with $T_{\max} = 1$:

$$\sum_{j_1\dots j_N=1}^{3} T^2_{j_1\dots j_N} > 1 \quad \Longrightarrow \quad T \text{ is entangled.} \tag{11}$$

First note that this holds for any quantum state, pure or mixed. The criterion is therefore state independent, in contradistinction to linear entanglement witnesses which are typically tailored to certain states. Furthermore, to prove entanglement it is sufficient to break the threshold on the right hand side, i.e. in general it is not necessary to measure all correlations. Using fundamental properties of the correlation tensor, we designed schemes to minimise the number of required correlation measurements [E,30].

In Ref. [E] we proposed and implemented a scheme based on the Schmidt decomposition of a pure quantum state. In this context Schmidt decomposition implies that a measurement along local Bloch vectors of a quantum state reveals the highest correlation in a state. Therefore, this is a natural starting point when verifying entanglement using (11). For the states which have vanishing local Bloch vectors we propose a filtering scheme which with one additional correlation measurement reveals maximal correlations in a state. With the help of correlation complementarity we also designed efficient algorithms for entanglement detection in unknown states. The idea is that if big correlations are measured, there is no need to measure all anti-commuting observables as they must have small expectation values and so we can directly move to sectors of the correlation tensor where there is still a chance of large correlation.

One more advantage of many of our conditions is that they sum up squared correlations, i.e. with further measurements we always increase the left-hand side and we can stop measuring as soon as the threshold is exceeded. This is in contrast with linear entanglement witnesses where all the measurements defining it have to be done as intermediate measurements could subtract from the corresponding left hand side. Ref. [H] gives an explicit example of such subtraction as well as methods of finding metric G given a state of interest.

Finally, we used these techniques together with our conditions for violation of Bell inequalities [3,4] to study the interplay between entanglement and local realism [C]. We verified how entanglement and ability to violate certain classes of Bell inequalities depends on noises applied to individual qubits. The work is very systematic: we study effects of white noise, coloured noise, depolarising, dephasing, and amplitude damping on multi-qubit systems prepared in Bell states, GHZ states, generalised GHZ, and W states. In all these scenarios one can find noise strengths for which entangled states satisfy broad classes of Bell inequalities. In particular, there are examples where this happens for infinitely many qubits.

Entanglement without correlations. Correlations between measurement results are the most prominent feature of entanglement. Every entangled bipartite state gives rise to non-vanishing elements of the correlation tensor. However, extrapolation of this result to multipartite systems is no longer true [Kaszlikowski2008]. There exist genuinely N-partite entangled states which nevertheless have no N-party correlations.

In Ref. [K] we showed a simple construction of no correlation states. For any pure quantum state $|\psi\rangle$ we constructed a state $|\bar{\psi}\rangle$ which has exactly opposite all correlations between an odd number of observers. We proved it by showing that $|\bar{\psi}\rangle$ can be mathematically obtained from $|\psi\rangle$ by application of local universal-not gates. These gates reverse the eigenvalues of all Pauli operators and therefore application of an odd number of them reverses the correlations. Importantly, although universal-not gate is not a unitary operation, we showed that $|\bar{\psi}\rangle$ is always a proper physical state. As such we can mix it with the original state

$$\rho_{\mathrm{nc}} = \frac{1}{2} |\psi\rangle \langle\psi| + \frac{1}{2} |\bar{\psi}\rangle \langle\bar{\psi}| , \tag{12}$$

and whenever the total number of qubits is odd we obtain the promised no correlation state. Next, we presented infinite families of such states which are additionally genuinely multiparty entangled. In order to complete the proof we again utilised our criteria for entanglement detection described above. These new states were realised experimentally and a careful error analysis confirmed entanglement without correlations.

5. Discord

As for the quantum entanglement, there are many measures proposed that quantify non-classical correlations present in some separable states and they could all be broadly called "discord" from the name of one of the first such measures [Ollivier2001]. They are reviewed in Ref. [G] and importantly essentially all of them set the same boundary between classical and quantum correlations. As classical they treat correlations in states that are invariant under local projective measurements. For bipartite systems, if local measurement is consider only on one side this notion corresponds to so-called classical-quantum states:

$$\rho_{\mathrm{cq}} = \sum_i p_i |i\rangle \langle i| \otimes \rho_i, \tag{13}$$

where p_i's are probabilities, $\{|i\rangle\}$ form orthonormal basis and ρ_i are arbitrary. Similarly, if both parties make measurements, as classical one treats classical-classical states:

$$\rho_{\mathrm{cc}} = \sum_{i,j} p_{ij} |i\rangle \langle i| \otimes |j\rangle \langle j| , \tag{14}$$

where now both local bases are orthonormal, and p_{ij} is the joint probability distribution. The latter notion is probably the closest to what one would at first glance describe as classical correlations because it encompasses two classical random variables, i and j, just written in quantum formalism.

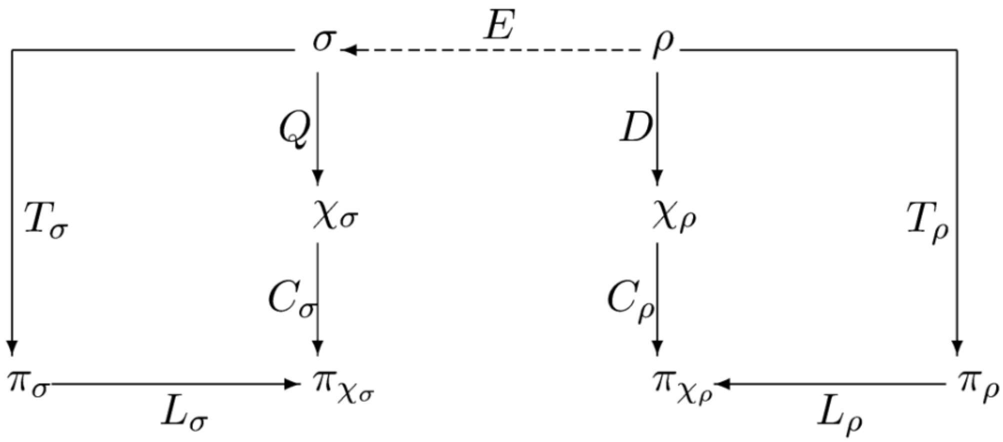

FIG. 2 Correlations in a quantum state. An arrow from x to y, i.e. $x \to y$, indicates that y is the closest state to x as measured by the relative entropy $S(x||y)$. The state ρ in general belongs to a set of entangled states $\mathcal{E}$, the state σ belongs to the set of separable states $\mathcal{S}$, the state χ belongs to the set of classical states $\mathcal{C}$ (either classical-classical or classical-quantum states), and π belongs to the set of product states $\mathcal{P}$. The distances are: entanglement E, quantum discord D, quantum dissonance Q, total mutual information T_ρ and T_σ, and classical correlations C_ρ and C_σ. All relative entropies, except for entanglement, reduce to the differences in entropies of y and x.

Unified view of correlations. Given so many measures of quantum discord as well as entanglement, it is difficult to compare between them. In Ref. [B] we remedy this by introducing a single framework for various correlations. At the time of writing the article also provided one of the first tools to tackle discord in multipartite systems. Our measures of correlations are based on the idea that a distance from a given state to the closest state without the desired property (e.g., entanglement or discord) is a measure of that property. For example, the distance to the closest separable state is a meaningful measure of entanglement. If the distance is measured with relative entropy, the resulting measure of entanglement is the relative entropy of entanglement [Vedral1997,Vedral1998]. We also used relative entropy to define measures of nonclassical correlations, though many other distance measures can serve just as well. Since all the distances are measured with relative entropy, this provides a consistent way to compare different correlations, e.g. entanglement, discord, classical correlations.

The relative entropy between two quantum states x and y is defined as

$$S(x||y) = \mathrm{Tr}(x \log x) - \mathrm{Tr}(x \log y). \tag{15}$$

The relative entropy is a non-negative quantity and due to this property it often appears in the context of distance measure though technically it is not a distance; e.g., it is not symmetric. In our approach, see Fig. 2, one starts with a state ρ. T_ρ is the total mutual information of ρ given by the distance to the closest product state. It captures all the correlations in a quantum state. If ρ is entangled, its entanglement is measured by the relative entropy of entanglement, E, which is the distance to the closest separable state σ. Having found σ, one then finds the closest classical state, χ_σ, to it. This distance, denoted by Q, contains the rest of nonclassical correlations (it is similar to discord [Henderson2001, Ollivier2001] but entanglement is excluded). We call this quantity quantum dissonance. Alternatively, if we are interested in quantity similar to original discord, here denoted by D, then we find the distance between ρ and closest classical state χ_ρ. Summing up,

FIG. 3 The simplest communication scenario. We study what influence exchange of particle C has on correlations, information, entanglement etc. between the laboratories.

we have the following nonclassical correlations:

$$E = \min_{\sigma\in\mathcal{S}} S(\rho||\sigma) \qquad \text{(entanglement)}, \tag{16}$$
$$D = \min_{\chi\in\mathcal{C}} S(\rho||\chi) \qquad \text{(discord)}, \tag{17}$$
$$Q = \min_{\chi\in\mathcal{C}} S(\sigma||\chi) \qquad \text{(dissonance)}. \tag{18}$$

Finally we compute classical correlations as the minimal distance between a classically correlated state, χ, and a product state π:

$$C = \min_{\pi\in\mathcal{P}} S(\chi||\pi) \qquad \text{(classical correlations)}. \tag{19}$$

Next we proved that discord and dissonance reduce to calculation of entropic costs of measurement and therefore discord D is equivalent to the zero-way or one-way quantum deficit introduced by the Horodecki family and collaborators [Oppenheim2002, Horodecki2003, Horodecki2005a]:

$$D = \min_{\Pi=\Pi_1\otimes\cdots\otimes\Pi_N} S(\Pi(\rho)) - S(\rho), \tag{20}$$
$$Q = \min_{\Pi=\Pi_1\otimes\cdots\otimes\Pi_N} S(\Pi(\sigma)) - S(\sigma), \tag{21}$$

where Π denotes a projective measurement on individual subsystems. Depending on whether we are interested in the set of classical-classical or classical-quantum states as classical states we choose the measurements on suitable subsystems. This choice also decides whether D is equal to the zero-way or one-way deficit.

With all these quantities at hand we ask what are their consequences. In Ref. [B] we noted that dissonance can be present even in pure entangled states, but only multipartite. For example, the state $|W\rangle = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle)$, has the closest separable state [Wei2004]: $\sigma = \frac{8}{27}|000\rangle\langle 000| + \frac{12}{27}|W\rangle\langle W| + \frac{6}{27}|\bar{W}\rangle\langle\bar{W}| + \frac{1}{27}|111\rangle\langle 111|$, where $|\bar{W}\rangle = \frac{1}{\sqrt{3}}(|011\rangle + |101\rangle + |110\rangle)$. It turns out that this state contains a lot of nonclassical correlations in form of quantum dissonance (almost 1 bit).

Entanglement distribution. A striking example of usefulness of quantum discord comes from considering quantum entanglement and general communication scenarios. Communication is the exchange of physical systems aimed at establishing correlations between communicating parties. Conceptually the most basic communication scenario is depicted in Fig. 3. It is then simple to show that the gain in mutual information caused by the exchange of particle C, i.e. $I_{A:CB} - I_{AC:B}$, is bounded by the communicated information, $I_{AB:C}$ [F]:

$$I_{A:CB} - I_{AC:B} \leq I_{AB:C}. \tag{22}$$

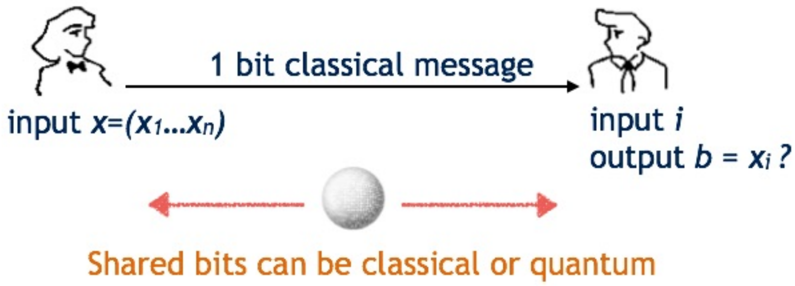

FIG. 4 $n \to 1$ random access codes with shared randomness. Alice and Bob share a finite number of classical or quantum bits from a common source (shared randomness). Alice is allowed to send a single classical bit to Bob, who tries to guess the ith bit, x_i, of Alice's input string.

In particular, this shows that no communication causes no information gain. This seems so basic that one could expect that every kind of reasonable correlations satisfies a similar constraint. Since no doubt quantum entanglement is a reasonable type of correlations, e.g. it empowers quantum teleportation [Bennett1993] or quantum cryptography [Ekert1991], we would expect it to satisfy condition similar to (22).

Surprisingly, [Cubitt2003] presented an example where entanglement is gained without it being communicated, i.e. $E_{A:CB} - E_{AC:B} > 0$, while $E_{AB:C} = 0$. This process is called entanglement distribution via separable states and it points to a question of what actually limits entanglement gain. In Ref. [F] we showed that it is the communicated quantum discord that provides a bound on entanglement gain (see also [Streltsov2012] for a parallel work):

$$E_{A:CB} - E_{AC:B} \leq D_{AB|C}. \tag{23}$$

Let us list main consequences of (23): (i) For zero-discord states there is no entanglement gain. This is just a statement that LOCC does not allow entanglement gain [Bennett1996]. Indeed, states of vanishing discord are of the form $\sum_j p_j \rho_j^{AB} \otimes |j\rangle\langle j|$, and therefore comunication C embodies classical information. In this context, Eq. (23) is a generalisation of monotonicity of entanglement under LOCC to the case of quantum communication. (ii) It generalises subadditivity of entropy. We proved that not only entanglement gain but also entanglement decay is bounded by the discord, i.e. $|E_{A:CB} - E_{AC:B}| \leq D_{AB|C}$. For pure states this reduces to $|S_A - S_B| \leq S_{AB}$, where S_j is the entropy of the jth subsystem. The later is known as the Araki-Lieb inequality [Araki1970] and is equivalent to the subadditivity of entropy for subsystems AC and BC. Accordingly, Eq. (23) can be seen as a generalisation of the subadditivity of entropy valid for tripartite mixed states. (iii) It gives meaning to a negative conditional entropy [Horodecki2005b]. Consider a bipartite system ρ_{AC} with negative conditional entropy $S_{C|A} = S_{AC} - S_A$. There always exists a pure tripartite state $|\psi_{ABC}\rangle$ which has ρ_{AC} as subsystem. We place the purifying particle B in a distant laboratory and note that the left hand side of (23) is now given by $E_{A:CB} - E_{AC:B} = -S_{C|A}$. Therefore the negative entropy gives entanglement gain caused by communication of C to the purifying laboratory.

Apart from fundamental interest, the distribution of entanglement via separable carriers is also helpful in the case of noisy communication and noisy laboratories. We showed this in Ref. [I] where this protocol was implemented for the first time using qubits encoded in polarisation of photons and parallel works [Vollmer2013,Peuntinger2013] demonstrated this way of entanglement distribution with continuous variables.

Random access codes. Our last example of usefulness of discord are problems that use finite amount of shared randomness. The general argument runs as follows. It is known from Bell's theorem that quantum predictions for some entangled states cannot be mimicked using local hidden variable (LHV) models. From a computer science perspective, LHV models may be interpreted as classical computers operating on a potentially infinite number of correlated bits originating from a common source. As such, Bell inequality violations achieved through entangled states are able to characterise the quantum advantage of certain tasks, so long as the task itself imposes no restriction on the availability of correlated bits. However, if the number of shared bits is limited, additional constraints are placed on the possible LHV models, and separable states may become a useful resource. Bell violations are therefore no longer necessary to achieve a quantum advantage.

In Ref. [J] we showed this explicitly for the task called random access code. Imagine that Bob would like to know (better than just by sheer guess) a random number from Alice's telephone book. Is it necessary for Alice to send Bob the whole book? Or can she communicate fewer "encoded" pages such that Bob is reasonably confident of getting the correct number? Random access codes are strategies designed to solve this problem. As illustrated in Fig. 4 , in a classical $n \to 1$ random access code (RAC) Alice receives a random n-bit input x, and communicates a single bit to Bob, who given this piece of information tries to guess the ith bit of Alice, x_i, by outputting his guess b_i (in every run i is chosen at random). We may construct quantum versions of this task by either having Alice communicate a single quantum bit [Ambainis2002] or by having Alice and Bob share an entangled quantum state aided by a single bit of classical communication [Pawlowski2010]. We study here the latter version of the problem and allow for arbitrary quantum states in place of just entangled ones. The role of quantum discord in the former version of the problem was considered in [Yao2012]. Our choice makes the relevance of shared randomness more transparent because by restricting the communication to classical the only additional resources facilitating the process are the assisting (qu)bits.

A standard figure of merit characterizing the efficiency of the RAC is the probability $P_{\min}$ of Bob's correct guess in the worst-case scenario (minimised over x and i), i.e. $P_{\min} = \min_{x,i} \Pr(b_i = x_i)$. We showed that for two bits of randomness a classical $n \to 1$ RAC has: (i) $P_{\min} \leq \frac{1}{2}$ for $n > 2$; (ii) $P_{\min} \leq \frac{2}{3}$ for $n = 2$; (iii) $P_{\min} \leq \frac{1}{2}$ for all n, if the random bits have maximally mixed marginals for Bob. Interestingly, the best classical protocol utilises bits that are biased, i.e. the bias in shared randomness can be used to gain additional efficiency. This is in contrast with many studies of randomness which often employ so-called common randomness, i.e. pairs of perfectly correlated and locally completely random bits. We then presented quantum codes assisted with Bell diagonal states and showed, for example, that $3 \to 1$ code has efficincy:

$$P_{\min} = \frac{1}{2}\left(1 + \frac{1}{\sqrt{T_1^{-2} + T_2^{-2} + T_3^{-2}}}\right), \tag{24}$$

where T_j are the diagonal elements of the correlation tensor of the Bell diagonal state (in the derivation it is assumed that all T_js are not zero). As $P_{\min} > \frac{1}{2}$, this quantum code is thus more efficient than the best classical code. If one restricts themselves to separable Bell diagonal states, it can be shown that $P_{\min}$ in this case is as high as $P_{\min} = \frac{1}{2}(1 + \frac{1}{3\sqrt{3}}) \approx 0.596$, considerably above the classical bound. Furthermore, examples exist where separable states outperform some entangled ones.

Finally we would like to give an argument suggesting that many sensible discord-like measures are still to be discovered. All of them set the same boundary between classical and quantum correlations, but the actual amount of nonclassical correlations is of course measure dependent and we might expect yet other quantities whose values are better suited to capture performance of various tasks. This is nicely illustrated by random access codes studied here. Although quantum discord empowers quantum advantage in our examples and furthermore it is proportional to the efficiency of the protocol for fixed classes of states, it should be noted that the amount of the geometric quantum discord for different classes of states [Dakic2010] is not an indicator of the usefulness of the states for quantum random access codes. Namely, our optimal separable state for $2 \to 1$ code has geometric discord $D_{\text{sep}} = \frac{1}{2\sqrt{2}}$ that corresponds to $P_{\min} = \frac{1}{2}(1 + D_{\text{sep}})$. For the Werner states $D_{\text{Wer}} = p$, where p gives the admixture of entanglement, and this corresponds to $P_{\min} = \frac{1}{2}(1 + D_{\text{Wer}}/\sqrt{2})$. Therefore, Werner states containing more discord than the separable state, i.e. for $D_{\text{Wer}} \in (\frac{1}{2\sqrt{2}}, \frac{1}{2})$, still give worse $P_{\min}$ than the separable state. The precise physical quantity that is a resource for better quantum codes is at present unknown, but it will be yet another measure from the series that we broadly called "discord".

V. OTHER SCIENTIFIC ACHIEVEMENTS

Bibliometric data:

- number of publications: **45** (**36** after Ph.D)
- total number of citations: **1500** (**1450** without self-citations)
- H-index: **15**
- total impact factor: **300**

A. before Ph.D

1. New Bell-type inequalities and their applications

We derived tight Bell's inequalities for multiple observers each choosing between multiple settings [3,4,6]. For some of them we also derived necessary and sufficient conditions for their violation by quantum predictions. Their link with communication complexity problems is studied in detail in Refs. [1,6].

2. Assumptions behind Bell's theorem

We clarified the role of locality in certain attempts to prove Bell's theorem without it [2]. We showed that to certain extend one can relax assumption on experimenter's freedom in Bell's theorem and this has consequences to cryptography allowing eavesdroppers to partially know the settings of legitimate users [5]. We extended the theorem by Leggett on non-local hidden variables [Leggett2003] and verified experimentally that a broad class of such models cannot describe nature [7,9].

3. Measurements on composite qudits

We showed how to measure generalised Pauli operators on multiple component systems and applied it to quantum cryptography [8].

B. after Ph.D

1. Mutually unbiased bases

We studied relations between the number of mutually unbiased bases in Hilbert space of dimension d and the number of orthogonal Latin squares of border d. We proposed a concrete map from squares to the bases

[11,18], which however turns out not to related all the squares to the bases in certain composite dimensions [15]. For composite dimensions we proved that the amount of entanglement contained is any set of $d+1$ such bases (if they exist) is always the same [21].

2. Hidden variable models

We showed that in order to explain quantum statistics of the results of n measurements, it is sufficient to take a hidden variable model with polynomial in n number of hidden variable states, i.e. collections of vectors which contain predetermined results for all n different measurements [10]. In Ref. [16] we proved that a hidden variable model has to provide information about distant setting and outcome in order to allow for Bell inequality violation. While information about the outcome can be encoded in the shared hidden variables, information about the setting needs to be non-locally transferred within these models. Ref. [19] shows how to violate a Bell inequality in the presence of super-selection rules. It turns out that the reference frames need to be prepared jointly. A curious effect is presented in Ref. [26] and was effectively observed in our experiment [K]. We show a quantum state which admits explicit local hidden variable models for correlations between any fixed number of observers. However, these models are incompatible as it is possible to violate a Bell inequality which combines correlations between different number of observers. Finally, in Ref. [34] we provide evidence that already two qubits can be prepared in a mixed entangled quantum state which admits a local hidden variable model for all non-trivial admixtures of noise (previous examples used infinitely dimensional systems).

3. Quantum foundations

In Ref. [13] we introduce a principle of information causality, which states that information transfer of n classical bits can cause information gain of at most n bits (even if methods used to read different remote bits are different). This is then shown to be responsible for the quantum Tsirelson bound. In [14] we propose a link between logical independence (a proposition is independent of the axioms if it can neither be proved nor disproved from the axioms) and quantum randomness. We demonstrate how to encode axioms in quantum states and how to encode propositions in quantum measurements. It turns out that whenever a proposition is logically independent, the measurement results are random. Ref. [17] introduces a hierarchy of models where physical systems have limited information content. This is shown to lead to complementary measurements and define computational abilities of these systems. The hierarchy includes classical, quantum, as well as some generalised probabilistic theories. Ref. [20] studies experimentally information distribution in various two-qubit states. In [31] we construct a model which predicts genuine triple-slit interference [Sorkin1994]. This model reduces to standard quantum mechanics if certain parameters vanish. Since quantum mechanics does not allow for the genuine triple-slit interference we hope the model will shed light why this is the case. Finally, in Ref. [32] we showed that contextuality, non-locality and temporal Bell inequalities are different faces of the same coin. All of them are about existence of certain joint probability distributions.

4. Quantum to classical transition

We show that measurements of quantities similar to magnetisation on macroscopic bodies can practically always be described by a classical-like model [22].

5. Quantum computing

In Ref. [23] we propose efficient method to generate polarisation-entangled NOON states. They are known to be a resource for quantum lithography and quantum metrology. In [33] we prove that multi-point temporal quantum correlations are a resource for universal one-way quantum computing [Briegel2001].

6. Quantum correlations

We show how to use our method [A] to detect genuine multi-qubit entanglement [24] as well as how to detect it with only bipartite correlations [28]. Extension of Ref. [E] that contains more details and studies in more depth multipartite systems is given in Ref. [30]. In Ref. [25] we prove and experimentally demonstrate that discord empowers a version of the remote state preparation (see also [Horodecki2014]).

7. Quantum biology

We studied the role quantum coherence might play in the avian compass in Refs. [27,29].

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