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Structure of the set of positive maps between low dimensional matrix algebras

Tomasz Ignacy Tylec

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Abstract

Convex cone of positive maps plays the crucial role in physics (theory of dynamical systems), theory of quantum information (classification of entangled states) and in mathematics (e.g. in the theory of operator algebras). From the physical point of view, the convex subset of positive unital maps is of special interest. This work is mainly devoted to the description of this set.

The thesis starts with general introduction to the theory of tensor products of Banach spaces. This language allows us to formulate the isometric generalisation of Choi-Jamiołkowski isomorphism (which works also for infinite dimensional operator algebras). We formulate there few corollaries and propositions, which were not previously published, but the content of the first part of the thesis is a result of straightforward application of the results found in classical work of Grothendieck *Produits tensoriels topologiques et espaces nucléaires.*

The isometric property of generalised Choi-Jamiołkowski isomorphism is fundamental, as it mean that the isomorphism preserves geometry of sets. On the basis of this remark, in the second part of the thesis, we analyse important geometric properties of the set of positive unital maps between matrix algebras $M_n(\mathbb{C}) \to M_n(\mathbb{C})$.

Firstly, we identify the subset $\mathfrak{D}_n \subset M_n(\mathbb{C}) \to M_n(\mathbb{C})$ of Choi matrices corresponding to positive unital maps. Some general properties of the elements of $\mathfrak{D}_n$ are discussed. We also analyse partition of $\mathfrak{D}_n$ which is induced by the properties of dual maps of corresponding positive maps.

Secondly, we attempt to find all exposed positive unital maps in $\mathfrak{D}_2$. We identified a large class of such maps, but we we were also able to show, that the remaining ones are still dense in the set of extreme maps. We also indicate how more complex the case of $\mathfrak{D}_n$ for $n \geq 3$ is.

Thirdly, we deeply analyse properties of the decomposition of Choi matrix of Choi map [Lin. Alg. Appl. 10, 285 (1975)]. This decomposition was introduced in [J. Math. Phys. 54, 073508 (2013)]. We discuss connection of this decomposition with other elements of the family of generalised Choi maps defined in [Lin. Alg. Appl. 171, 213 (1992)]. We also propose scheme that allows to generalise this family to maps $M_n(\mathbb{C}) \to M_n(\mathbb{C})$ for $n \geq 4$.

Finally, we discuss selfadjoint unitary elements in $\mathfrak{D}_n$. For $n = 2, 3$ such elements correspond to maps that are unitarily equivalent to the transposition map. For $n \geq 4$ we do not have conclusive results, but we show that direct generalisation of the proof in $n = 2, 3$ case is impossible, and we provide strong arguments in favour of the hypothesis that in $\mathfrak{D}_9$ there are selfadjoint unitary elements, which correspond to maps that are not unitarily equivalent to the transposition map. We close the thesis with short discussion of elements of $\mathfrak{D}_n$ which are selfadjoint partial isometries.

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