ABSTRACT OF THE DOCTORAL THESIS

titled: **„The use of Katětov order in the study of topological spaces and ultrafilters "**

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In the dissertation, we dealt with thirteen selected ideals and investigated their basic properties. We also obtained general theorems characterizing all ideals with certain properties. We were particularly interested in properties related to ideal convergence and the order structure of ideals. The main theme of this dissertation is the Katětov order named after the Czech mathematician Miroslav Katětov, which we used in the study of topological spaces and ultrafilters. Our research was inspired by the work of Brendle, Flaškova and Kojman.

For several ideals related to combinatorial theorems (e.g., Hindman, van der Waerden, Brown, Folkman), we considered ideal convergence by investigating the properties BW, hBW, FinBW, hFinBW introduced in the paper [21] by Filipów, Mrożek, Recław and Szuca. We then examined whether the ideals we presented are homogeneous. The concept of homogeneity of ideals was introduced by Kwela and Tryba and they gave a characterization of homogeneous ideals in the paper [54]. We used this characterization to show that some of the ideals we presented are homogeneous. At the end of the second chapter, we presented a graph showing the inclusions between the ideals in question (the only unknown remains the Erdős-Turán hypothesis).

In the third chapter, we were interested in the types of ultrafilters known so far, such as selective ultrafilters, P-points, or Q-points. They were dealt with by such well-known mathematicians as Baumgartner, Brendle, Hrušák, among others. We have introduced a new order and two cardinal coefficients, which were used to write the most important theorem in this chapter giving a characterization by means of the Katětov order for the existence of an $\mathcal{I}$-ultrafilter which is not a $\mathcal{J}$-ultrafilter. We have also distinguished between some types of $\mathcal{I}$-ultrafilters. For example, we have shown that consistently there is a $\mathcal{D}_{fin}$-ultrafilter, which is not a Q-point [53]. At the end of the chapter, we use the obtained results to show that some ideals are not homogeneous.

The fourth chapter deals with topological spaces such as differentially compact, Hindman, van der Waerden, Folkman. In the works [50, 49, 51, 24, 63, 27, 46] these spaces were dealt with by the following mathematicians: Filipów, Flašková, Jones, Kojman, Shelah, Shi. We strengthened Shi's results on differentially compact spaces by replacing the assumption of the continuum hypothesis by Martin's axiom. We have investigated how, using the Katětov order, we can distinguish given two classes of spaces or how to show inclusion between them. As an application of the general results, we showed that, under the assumption of the continuum hypothesis, there exists a Hindman space which is not an $\mathcal{I}_{\frac{1}{n}}$-space.

In the last chapter, we considered the ideals from chapter one and their relations in orders (we added the Rudin-Keisler and Rudin-Blass orders). We checked those relations that could not be achieved by the previous considerations and that were not known before. The work is summarized in a table in which we have gathered the results known so far and obtained in this dissertation. There we presented how the ideals we considered relate to each other in the given orders.